NEW SAT MATH PROBLEMS
arranged by Topic
and Difficulty Level

By Dr. Steve Warner

For the Revised SAT March 2016 and Beyond
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his book is for the revised SAT beginning in March 2016. If you are preparing for an SAT being administered before this date, then this is not the right book for you. The PSAT being given in October 2015 will have the new format, so you can use this book to prepare for that test, especially if you are going for a national merit scholarship.

There are many ways that a student can prepare for the SAT. But not all preparation is created equal. I always teach my students the methods that will give them the maximum result with the minimum amount of effort.

The book you are now reading is self-contained. Each problem was carefully created to ensure that you are making the most effective use of your time while preparing for the SAT. By grouping the problems given here by level and topic I have ensured that you can focus on the types of problems that will be most effective to improving your score.

1. Using this book effectively
   - Begin studying at least three months before the SAT
   - Practice SAT math problems twenty minutes each day
   - Choose a consistent study time and location

You will retain much more of what you study if you study in short bursts rather than if you try to tackle everything at once. So try to choose about a twenty minute block of time that you will dedicate to SAT math each day. Make it a habit. The results are well worth this small time commitment.

   - Every time you get a question wrong, mark it off, no matter what your mistake.
   - Begin each study session by first redoing problems from previous study sessions that you have marked off.
   - If you get a problem wrong again, keep it marked off.
Note that this book often emphasizes solving each problem in more than one way. Please listen to this advice. The same question is not generally repeated on any SAT so the important thing is learning as many techniques as possible.

Being able to solve any specific problem is of minimal importance. The more ways you have to solve a single problem the more prepared you will be to tackle a problem you have never seen before, and the quicker you will be able to solve that problem. Also, if you have multiple methods for solving a single problem, then on the actual SAT when you “check over” your work you will be able to redo each problem in a different way. This will eliminate all “careless” errors on the actual exam. Note that in this book the quickest solution to any problem will always be marked with an asterisk (*).

2. The magical mixture for success
A combination of three components will maximize your SAT math score with the least amount of effort.

- Learning test taking strategies that work specifically for standardized tests.
- Practicing SAT problems for a small amount of time each day for about three months before the SAT.
- Taking about four practice tests before test day to make sure you are applying the strategies effectively under timed conditions.

I will discuss each of these three components in a bit more detail.

Strategy: The more SAT specific strategies that you know the better off you will be. Throughout this book you will see many strategies being used. Some examples of basic strategies are “plugging in answer choices,” “taking guesses,” and “picking numbers.” Some more advanced strategies include “trying a simple operation,” and “moving the sides of a figure around.” Pay careful attention to as many strategies as possible and try to internalize them. Even if you do not need to use a strategy for that specific problem, you will certainly find it useful for other problems in the future.

Practice: The problems given in this book, together with the problems in the practice tests from the College Board’s Official Study Guide (2016 Edition), are more than enough to vastly improve your current SAT math score. All you need to do is work on these problems for about ten to twenty minutes each day over a period of three to four months and the final result will far exceed your expectations.
Let me further break this component into two subcomponents – topic and level.

**Topic:** You want to practice each of the four general math topics given on the SAT and improve in each independently. The four topics are **Heart of Algebra, Geometry and Trig, Passport to Advanced Math**, and **Problem Solving and Data Analysis**. The problem sets in this book are broken into these four topics.

**Level:** You will make the best use of your time by primarily practicing problems that are at and slightly above your current ability level. For example, if you are struggling with Level 2 Geometry and Trig problems, then it makes no sense at all to practice Level 5 Geometry and Trig problems. Keep working on Level 2 until you are comfortable, and then slowly move up to Level 3. Maybe you should never attempt those Level 5 problems. You can get an exceptional score without them (higher than a 700).

**Tests:** You want to take about four practice tests before test day to make sure that you are implementing strategies correctly and using your time wisely under pressure. For this task you should use “The Official SAT Study Guide (2016 Edition).” Take one test every few weeks to make sure that you are implementing all the strategies you have learned correctly under timed conditions.

### 3. Practice problems of the appropriate level

Roughly speaking about one third of the math problems on the SAT are easy, one third are medium, and one third are hard. If you answer two thirds of the math questions on the SAT correctly, then your score will be approximately a 600 (out of 800). That’s right—you can get about a 600 on the math portion of the SAT without answering a single hard question.

Keep track of your current ability level so that you know the types of problems you should focus on. If you are currently scoring around a 400 on your practice tests, then you should be focusing primarily on Level 1, 2, and 3 problems. You can easily raise your score 100 points without having to practice a single hard problem.

If you are currently scoring about a 500, then your primary focus should be Level 2 and 3, but you should also do some Level 1 and 4 problems.

If you are scoring around a 600, you should be focusing on Level 2, 3, and 4 problems, but you should do some Level 1 and 5 problems as well.

Those of you at the 700 level really need to focus on those Level 4 and 5 problems.
If you really want to refine your studying, then you should keep track of your ability level in each of the four major categories of problems:

- Heart of Algebra
- Geometry and Trig
- Passport to Advanced Math
- Problem Solving and Data Analysis

For example, many students have trouble with very easy Geometry and Trig problems, even though they can do more difficult algebra problems. This type of student may want to focus on Level 1, 2, and 3 Geometry and Trig questions, but Level 3 and 4 Heart of Algebra questions.

4. Practice in small amounts over a long period of time

Ideally you want to practice doing SAT math problems ten to twenty minutes each day beginning at least 3 months before the exam. You will retain much more of what you study if you study in short bursts than if you try to tackle everything at once.

The only exception is on a day you do a practice test. You should do at least four practice tests before you take the SAT. Ideally you should do your practice tests on a Saturday or Sunday morning. At first you can do just the math sections. The last one or two times you take a practice test you should do the whole test in one sitting. As tedious as this is, it will prepare you for the amount of endurance that it will take to get through this exam.

So try to choose about a twenty minute block of time that you will dedicate to SAT math every night. Make it a habit. The results are well worth this small time commitment.

5. Redo the problems you get wrong over and over and over until you get them right

If you get a problem wrong, and never attempt the problem again, then it is extremely unlikely that you will get a similar problem correct if it appears on the SAT.

Most students will read an explanation of the solution, or have someone explain it to them, and then never look at the problem again. This is not how you optimize your SAT score. To be sure that you will get a similar problem correct on the SAT, you must get the problem correct before the SAT—and without actually remembering the problem.
This means that after getting a problem incorrect, you should go over and understand why you got it wrong, wait at least a few days, then attempt the same problem again. If you get it right you can cross it off your list of problems to review. If you get it wrong, keep revisiting it every few days until you get it right. Your score does not improve by getting problems correct. **Your score improves when you learn from your mistakes.**

6. **Check your answers properly**
When you go back to check your earlier answers for careless errors do not simply look over your work to try to catch a mistake. This is usually a waste of time. Always redo the problem without looking at any of your previous work. Ideally, you want to use a different method than you used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you do not know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers do not match up, then you know that this a problem you need to spend a little more time on to figure out where your error is. This may seem time consuming, but that’s okay. It is better to spend more time checking over a few problems than to rush through a lot of problems and repeat the same mistakes.

7. **Take a guess whenever you cannot solve a problem**
There is no guessing penalty on the SAT. Whenever you do not know how to solve a problem take a guess. Ideally you should eliminate as many answer choices as possible before taking your guess, but if you have no idea whatsoever do not waste time overthinking. Simply put down an answer and move on. You should certainly mark it off and come back to it later if you have time.

8. **Pace yourself**
Do not waste your time on a question that is too hard or will take too long. After you’ve been working on a question for about 30 to 45 seconds you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still do not know how to do the problem or you are using a technique that is going to take a long time, mark it off and come back to it later if you have time.
If you do not know the correct answer, eliminate as many answer choices as you can and take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Do not sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

9. **Attempt the right number of questions**

Many students make the mistake of thinking that they have to attempt every single SAT math question when they are taking the test. There is no such rule. In fact, most students will increase their SAT score by reducing the number of questions they attempt.

There are two math sections on the SAT – one where a calculator is allowed and one where a calculator is not allowed. The calculator section has 30 multiple choice (mc) questions and 8 free response (grid in) questions. The non-calculator section has 15 multiple choice (mc) questions and 5 free response (grid in) questions.

You should first make sure that you know what you got on your last SAT practice test, actual SAT, or actual PSAT (whichever you took last). What follows is a general goal you should go for when taking the exam.

<table>
<thead>
<tr>
<th>Score</th>
<th>MC (Calculator Allowed)</th>
<th>Grid In (Calculator Allowed)</th>
<th>MC (Calculator Not Allowed)</th>
<th>Grid In (Calculator Not Allowed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 330</td>
<td>10/30</td>
<td>3/8</td>
<td>4/15</td>
<td>1/5</td>
</tr>
<tr>
<td>330 – 370</td>
<td>15/30</td>
<td>4/8</td>
<td>6/15</td>
<td>2/5</td>
</tr>
<tr>
<td>380 – 430</td>
<td>18/30</td>
<td>5/8</td>
<td>8/15</td>
<td>2/5</td>
</tr>
<tr>
<td>440 – 490</td>
<td>21/30</td>
<td>6/8</td>
<td>9/15</td>
<td>3/5</td>
</tr>
<tr>
<td>500 – 550</td>
<td>24/30</td>
<td>6/8</td>
<td>11/15</td>
<td>4/5</td>
</tr>
<tr>
<td>560 – 620</td>
<td>27/30</td>
<td>7/8</td>
<td>13/15</td>
<td>4/5</td>
</tr>
<tr>
<td>630 – 800</td>
<td>30/30</td>
<td>8/8</td>
<td>15/15</td>
<td>5/5</td>
</tr>
</tbody>
</table>

For example, a student with a current score of 450 should attempt 21 multiple choice questions and 6 grid ins from the section where a calculator is allowed, and 9 multiple choice questions and 3 grid in questions from the section where a calculator is not allowed.
This is just a general guideline. Of course it can be fine-tuned. As a simple example, if you are particularly strong at Algebra problems, but very weak at Geometry and Trig problems, then you may want to try every Algebra problem no matter where it appears, and you may want to reduce the number of Geometry and Trig problems you attempt.

Remember that there is no guessing penalty on the SAT, so you should not leave any questions blank. This does not mean you should attempt every question. It means that if you are running out of time make sure you fill in answers for all the questions you did not have time to attempt.

10. Use your calculator wisely.
   - Use a TI-84 or comparable calculator if possible when practicing and during the SAT.
   - Make sure that your calculator has fresh batteries on test day.
   - You may have to switch between DEGREE and RADIANS modes during the test. If you are using a TI-84 (or equivalent) calculator press the MODE button and scroll down to the third line when necessary to switch between modes.

Below are the most important things you should practice on your graphing calculator.

   - Practice entering complicated computations in a single step.
   - Know when to insert parentheses:
     - Around numerators of fractions
     - Around denominators of fractions
     - Around exponents
     - Whenever you actually see parentheses in the expression

**Examples:**

We will substitute a 5 in for $x$ in each of the following examples.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Calculator computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{7x + 3}{2x - 11}$</td>
<td>$(7<em>5 + 3)/(2</em>5 - 11)$</td>
</tr>
<tr>
<td>$(3x - 8)^{2x-9}$</td>
<td>$(3<em>5 - 8)^{(2</em>5 - 9)}$</td>
</tr>
</tbody>
</table>

- Clear the screen before using it in a new problem. The big screen allows you to check over your computations easily.
- Press the ANS button (2ND (-) ) to use your last answer in the next computation.
• Press **2ND ENTER** to bring up your last computation for editing. This is especially useful when you are plugging in answer choices, or guessing and checking.

• You can press **2ND ENTER** over and over again to cycle backwards through all the computations you have ever done.

• Know where the $\sqrt{}$, $\pi$, and $^\wedge$ buttons are so you can reach them quickly.

• Change a decimal to a fraction by pressing **MATH ENTER ENTER**.

• Press the **MATH** button - in the first menu that appears you can take cube roots and $n$th roots for any $n$. Scroll right to **NUM** and you have $\text{lcm}(\text{ and } \text{gcd}(\text{. Scroll right to **PRB** and you have $\text{nPr}$, $\text{nCr}$, and $\text{!}$ to compute permutations, combinations and factorials very quickly.

• Know how to use the **SIN**, **COS** and **TAN** buttons as well as **SIN^{-1}**, **COS^{-1}** and **TAN^{-1}**.

You may find the following graphing tools useful.

• Press the **Y=** button to enter a function, and then hit **ZOOM 6** to graph it in a standard window.

• Practice using the **WINDOW** button to adjust the viewing window of your graph.

• Practice using the **TRACE** button to move along the graph and look at some of the points plotted.

• Pressing **2ND TRACE** (which is really **CALC**) will bring up a menu of useful items. For example selecting **ZERO** will tell you where the graph hits the $x$-axis, or equivalently where the function is zero. Selecting **MINIMUM** or **MAXIMUM** can find the vertex of a parabola. Selecting **INTERSECT** will find the point of intersection of 2 graphs.
11. Grid your answers correctly

The computer only grades what you have marked in the bubbles. The space above the bubbles is just for your convenience, and to help you do your bubbling correctly.

Never mark more than one circle in a column or the problem will automatically be marked wrong. You do not need to use all four columns. If you don’t use a column just leave it blank.

The symbols that you can grid in are the digits 0 through 9, a decimal point, and a division symbol for fractions. Note that there is no negative symbol. So answers to grid-ins cannot be negative. Also, there are only four slots, so you can’t get an answer such as 52,326.

Sometimes there is more than one correct answer to a grid-in question. Simply choose one of them to grid-in. Never try to fit more than one answer into the grid.

If your answer is a whole number such as 2451 or a decimal that only requires four or less slots such as 2.36, then simply enter the number starting at any column. The two examples just written must be started in the first column, but the number 16 can be entered starting in column 1, 2 or 3.

Note that there is no zero in column 1, so if your answer is 0 it must be gridded into column 2, 3 or 4.

Fractions can be gridded in any form as long as there are enough slots. The fraction 2/100 must be reduced to 1/50 simply because the first representation won’t fit in the grid.

Fractions can also be converted to decimals before being gridded in. If a decimal cannot fit in the grid, then you can simply truncate it to fit. But you must use every slot in this case. For example, the decimal .167777777… can be gridded as .167, but .16 or .17 would both be marked wrong.

Instead of truncating decimals you can also round them. For example, the decimal above could be gridded as .168. Truncating is preferred because there is no thinking involved and you are less likely to make a careless error.
Here are three ways to grid in the number 8/9.

*Never grid-in mixed numerals.* If your answer is $2\frac{1}{4}$, and you grid in the mixed numeral $2\frac{1}{4}$, then this will be read as $21/4$ and will be marked wrong. You must either grid in the decimal $2.25$ or the improper fraction $9/4$.

Here are two ways to grid in the mixed numeral $1\frac{1}{2}$ correctly.
1. Which of the following expressions is equivalent to $5a + 10b + 15c$?

   (A) $5(a + 2b + 3c)$
   (B) $5(a + 2b + 15c)$
   (C) $5(a + 10b + 15c)$
   (D) $5(a + 2b) + 3c$

**Solution by picking numbers:** Let’s choose values for $a$, $b$, and $c$, say $a = 2$, $b = 3$, $c = 4$. Then

$$5a + 10b + 15c = 5(2) + 10(3) + 15(4) = 10 + 30 + 60 = 100.$$  

Put a nice big dark circle around 100 so you can find it easier later. We now substitute $a = 2$, $b = 3$, $c = 4$ into each answer choice:

   (A) $5(2 + 2 \cdot 3 + 3 \cdot 4) = 100$
   (B) $5(2 + 2 \cdot 3 + 15 \cdot 4) = 340$
   (C) $5(2 + 10 \cdot 3 + 15 \cdot 4) = 460$
   (D) $5(2 + 2 \cdot 3) + 3 \cdot 4 = 52$

Since (B), (C), and (D) each came out incorrect, the answer is choice (A).

**Important note:** (A) is not the correct answer simply because it is equal to 100. It is correct because all three of the other choices are not 100. You absolutely must check all four choices!

**Remark:** All of the above computations can be done in a single step with your calculator (if a calculator is allowed for this problem).

**Notes about picking numbers:** (1) Observe that we picked a different number for each variable. We are less likely to get more than one answer choice to come out to the correct answer this way.
(2) We picked numbers that were simple, but not too simple. The number 2 is usually a good choice to start, if it is allowed. We then also picked 3 and 4 so that the numbers would be distinct (see note (1)).

(3) When using the strategy of picking numbers it is very important that we check every answer choice. It is possible for more than one choice to come out to the correct answer. We would then need to pick new numbers to try to eliminate all but one choice.

* Algebraic solution: We simply factor out a 5 to get

\[ 5a + 10b + 15c = 5(a + 2b + 3c) \]

This is choice (A).

Remarks: (1) If you have trouble seeing why the right hand side is the same as what we started with on the left, try working backwards and multiplying instead of factoring. In other words we have

\[ 5(a + 2b + 3c) = 5a + 10b + 15c \]

Note how the **distributive property** is being used here. Each term in parentheses is multiplied by the 5.

In general, the distributive property says that if \( x, y, \) and \( z \) are real numbers, then

\[ x(y + z) = xy + xz. \]

This property easily extends to expressions with more than two terms. For example,

\[ x(y + z + w) = xy + xz + xw. \]

(2) We can also solve this problem by starting with the answer choices and multiplying (as we did in Remark (1)) until we get \( 5a + 10b + 15c \).

2. Joseph joins a gym that charges $79.99 per month plus tax for a premium membership. A tax of 6% is applied to the monthly fee. Joseph is also charged a one-time initiation fee of $95 as soon as he joins. There is no contract so that Joseph can cancel at any time without having to pay a penalty. Which of the following represents Joseph’s total charge, in dollars, if he keeps his membership for \( t \) months?

(A) \( 1.06(79.99 + 95)t \)
(B) \( 1.06(79.99t + 95) \)
(C) \( 1.06(79.99t) + 95 \)
(D) \( (79.99 + .06t) + 95 \)
Solution by picking a number: (We will be using a calculator for this solution)

Let’s choose a value for $t$, say $t = 2$, so that Joseph keeps his gym membership for 2 months.

Now 6% of 79.99 is 4.80 (to the nearest cent). So each month of membership, including tax, is $79.99 + 4.80 = 84.79$ dollars. It follows that 2 months of membership, with tax, is $2 \cdot 84.79 = 169.58$ dollars. When we add the initiation fee we get $169.58 + 95 = \boxed{264.58}$ dollars.

Put a nice big, dark circle around the number $264.58$ so you can find it easily later. We now substitute $t = 2$ into each answer choice and use our calculator:

(A) $1.06(79.99 + 95) \cdot 2 \approx 370.98$

(B) $1.06(79.99 \cdot 2 + 95) \approx 270.28$

(C) $1.06(79.99 \cdot 2) + 95 \approx 264.58$

(D) $(79.99 + 0.06 \cdot 2) + 95 = 175.11$

Since choices (A), (B), and (D) came out incorrect, we can eliminate them. Therefore the answer is choice (C).

Important note: (C) is not the correct answer simply because it came out to 264.58. It is correct because all three of the other choices did not come out correct.

* Algebraic solution: Since the monthly membership fee is 79.99 dollars, and the tax is 6%, the total monthly fee, with tax, is $1.06(79.99)$ dollars per month. It follows that the total monthly fee for $t$ months is $1.06(79.99t)$. Finally, we add in the one-time initiation fee to get $1.06(79.99t) + 95$, choice (C).

Notes: (1) 6% can be written either as the decimal .06 or the fraction $\frac{6}{100}$.

To change a percent to a decimal, simply divide by 100, or equivalently, move the decimal point two places to the left, adding in zeros if necessary. Note that an integer has a “hidden” decimal point right after the number. In other words, 6 can be written as 6., so when we move the decimal point two places to the left we get .06 (we had to add in a zero as a placeholder).

To change a percent to a fraction, simply place the number in front of the percent symbol (%) over 100.
(2) Since the tax is 6%, it follows that the tax for $79.99 is \( \frac{6}{100} \times (79.99) \) dollars.

It follows that the total monthly fee, including tax, is

\[ 79.99 + \frac{6}{100} (79.99) \text{ dollars.} \]

We can use the distributive property to simplify this expression as follows:

\[ 79.99 + \frac{6}{100} (79.99) = 1(79.99) + \frac{6}{100} = 1.06(79.99) \]

(3) See problem 1 for more information on the distributive property.

(4) In note (2) we saw that one way to get the total monthly fee, including tax, is to add the amount of tax to the untaxed amount. A quicker way is to simply multiply the monthly fee by 1.06. A justification for why this works is given in the last line of note (2).

(5) If you need to pay a certain dollar amount more than once, simply multiply by the number of times you need to pay.

For example, if you need to pay 100 dollars five times, then the final result is that you pay \( 100 \cdot 5 = 500 \) dollars. More generally, if you need to pay 100 dollars \( t \) times, then the final result is that you pay \( 100t \) dollars.

In this problem we want to pay the monthly fee \( t \) times. Since the monthly fee is \( 1.06(79.99) \), the final result is \( 1.06(79.99)t \), or equivalently \( 1.06(79.99t) \).

(6) Don’t forget to add on the one-time initiation fee to \( 1.06(79.99t) \) to get \( 1.06(79.99t) + 95 \) dollars.

3. A high school has a $1000 budget to buy calculators. Each scientific calculator will cost the school $12.97 and each graphing calculator will cost the school $73.89. Which of the following inequalities represents the possible number of scientific calculators \( S \) and graphing calculators \( G \) that the school can purchase while staying within their specified budget?

\[
\begin{align*}
(A) & \quad 12.97S + 73.89G > 1000 \\
(B) & \quad 12.97S + 73.89G \leq 1000 \\
(C) & \quad \frac{12.97}{S} + \frac{73.89}{G} > 1000 \\
(D) & \quad \frac{12.97}{S} + \frac{73.89}{G} \leq 1000 
\end{align*}
\]
* **Algebraic solution:** The total cost, in dollars, for $S$ scientific calculators is $12.97S$, and the total cost, in dollars, for $G$ graphing calculators is $73.89G$.

It follows that the total cost, in dollars, for $S$ scientific calculators and $G$ graphing calculators is $12.97S + 73.89G$.

To stay within the school’s budget, we need this total cost to be less than or equal to 1000 dollars.

So the answer is $12.97S + 73.89G \leq 1000$, choice (B).

**Notes:**

1. When using the symbols “<” and “>”, the symbol always points to the smaller number (and similarly for “≤” and “≥”).

2. To stay within the specified budget means that the total must not exceed $1000$. Some equivalent ways to say this are as follows:
   - the total must not be greater than $1000$.
   - the total must be less than or equal to $1000$.
   - the total $T$ must satisfy $T \leq 1000$.

3. If the school were to spend exactly $1000$, they would still be within their budget. This is why the solution has “≤” instead of “<.”

4. If $-\frac{27}{10} < 2 - 5x < -\frac{13}{5}$, then give one possible value of $20x - 8$.

* **Solution by trying a simple operation:** Observe that

$$20x - 8 = 4(5x - 2) = -4(2 - 5x).$$

So we have

$$(-4) \left(-\frac{13}{5}\right) < -4(2 - 5x) < (-4)(-\frac{27}{10})$$

or equivalently

$$\frac{52}{5} < 20x - 8 < \frac{54}{5}$$

So we can grid in $\frac{53}{5}$.

**Notes:**

1. The simple operation we used here was multiplication by $-4$.

We simply multiplied each of the three parts of the given inequality by $-4$, noting that the inequalities reverse because we are multiplying by a negative number.
(2) Take careful note of how $-\frac{27}{10}$ and $-\frac{13}{5}$ changed positions when we multiplied by the negative number $-4$.

(3) If we are allowed to use a calculator for this problem we could multiply each of $-\frac{27}{10}$ and $-\frac{13}{5}$ by $-4$ in our calculator to get

$$(-4) \left(-\frac{27}{10}\right) = 10.8 \text{ and } (-4) \left(-\frac{13}{5}\right) = 10.4$$

So we can grid in 10.5, 10.6, or 10.7.

(4) We actually do not need to worry too much about the inequalities reversing in this problem. We can simply multiply each of $-\frac{27}{10}$ and $-\frac{13}{5}$ by $-4$, and then choose a number between the two numbers that we get.

5. The expression $3(5x + 8) − 4(3x − 2)$ is simplified to the form $ax + b$. What is the value of $ab$?

* **Algebraic solution:**

$$3(5x + 8) − 4(3x − 2) = 15x + 24 − 12x + 8 = 3x + 32.$$ 

So $a = 3$, $b = 32$, and therefore $ab = 3 \cdot 32 = 96$.

**Note:** Make sure you are using the distributive property correctly here.

For example $3(5x + 8) = 15x + 24$. A common mistake would be to write $3(5x + 8) = 15x + 8$.

Also, $-4(3x − 2) = −12x + 8$. A common mistake would be to write $-4(3x − 2) = −12x − 2$.

See problem 1 for more information on the distributive property.

6. If $x + 7y = 15$ and $x + 3y = 7$, what is the value of $x + 5y$?

* **Solution by trying a simple operation:** We add the two equations

$$x + 7y = 15$$
$$x + 3y = 7$$
$$2x + 10y = 22$$

Now observe that $2x + 10y = 2(x + 5y)$. So $x + 5y = \frac{22}{2} = 11$.

**Notes:** (1) We can also finish the problem by dividing each term of $2x + 10y = 22$ by 2.
We have $\frac{2x}{2} = x$, $\frac{10y}{2} = 5y$, and $\frac{22}{2} = 11$. So we get $\frac{2x}{2} + \frac{10y}{2} = \frac{22}{2}$, or equivalently $x + 5y = 11$.

(2) Although I do not recommend this for this problem, we could solve the system of equations for $x$ and $y$, and then substitute those values in for $x$ and $y$ in the expression $x + 5y$.

See problem 73 for several different ways to do this.

**Level 1: Geometry and Trig**

7. Given right triangle $\triangle PQR$ below, what is the length of $\overline{PQ}$?

![Diagram of right triangle $\triangle PQR$ with sides $PQ$, $QR$, and $PR$.]

(A) $\sqrt{2}$
(B) $\sqrt{5}$
(C) 5
(D) 7

*Solution using Pythagorean triples:* We use the Pythagorean triple 5, 12, 13 to see that $PQ = 5$, choice (C).

**Note:** The most common Pythagorean triples are 3, 4, 5 and 5, 12, 13. Two others that may come up are 8, 15, 17 and 7, 24, 25.

**Solution by the Pythagorean Theorem:** By the Pythagorean Theorem, we have $13^2 = (PQ)^2 + 12^2$. So $169 = (PQ)^2 + 144$. Subtracting 144 from each side of this equation yields $25 = (PQ)^2$, or $PQ = 5$, choice (C).

**Remarks:** (1) The Pythagorean Theorem says that if a right triangle has legs of lengths $a$ and $b$, and a hypotenuse of length $c$, then $c^2 = a^2 + b^2$.

(2) Be careful in this problem: the length of the hypotenuse is 13. So we replace $c$ by 13 in the Pythagorean Theorem.
(3) The equation \( x^2 = 25 \) would normally have two solutions: \( x = 5 \) and \( x = -5 \). But the length of a side of a triangle cannot be negative, so we reject \(-5\).

8. What is the radius of a circle whose circumference is \( \pi \)?

(A) \( \frac{1}{2} \)
(B) 1
(C) \( \frac{\pi}{2} \)
(D) \( \pi \)

**Solution by plugging in answer choices:** The circumference of a circle is \( C = 2\pi r \). Let’s start with choice (C) as our first guess. If \( r = \frac{\pi}{2} \), then \( C = 2\pi \left( \frac{\pi}{2} \right) = \pi^2 \). Since this is too big we can eliminate choices (C) and (D).

Let’s try choice (B) next. If \( r = 1 \), then \( C = 2\pi (1) = 2\pi \), still too big.

The answer must therefore be choice (A). Let’s verify this. If \( r = \frac{1}{2} \), then \( C = 2\pi \left( \frac{1}{2} \right) = \pi \). So the answer is indeed choice (A).

**Note:** When plugging in answer choices, it’s always a good idea to start with choice (B) or (C) unless there is a specific reason not to. In this problem, eliminating choice (C) allowed us to eliminate choice (D) as well, possibly saving us from having to do one extra computation.

**Algebraic solution:** We use the circumference formula \( C = 2\pi r \), and substitute \( \pi \) in for \( C \).

\[
C = 2\pi r \\
\pi = 2\pi r \\
\frac{\pi}{2\pi} = r \\
\frac{1}{2} = r
\]

This is choice (A).
9. In $\Delta PQR$ above, $\tan R = \frac{5}{12}$. What is the length of side $PR$?

(A) 11
(B) 13
(C) 15
(D) 16

* Since $\tan R = \frac{\text{OPP}}{\text{ADJ}}$, we have $\frac{5}{12} = \frac{\text{OPP}}{\text{ADJ}}$. Since the adjacent side is 12, the opposite side must be 5. So we have the following picture.

We now find $PR$ by using the Pythagorean Theorem, or better yet, recognizing the Pythagorean triple 5, 12, 13.

So $PR = 13$, choice (B).

Remarks: (1) If you don’t remember the Pythagorean triple 5, 12, 13, you can use the Pythagorean Theorem.

In this problem we have $c^2 = 5^2 + 12^2 = 169$. So $c = 13$.

(2) See problem 7 for more information about Pythagorean triples and the Pythagorean Theorem.

(3) The equation $c^2 = 169$ would normally have two solutions: $c = 13$ and $c = -13$. But the length of a side of a triangle cannot be negative, so we reject $-13$. 


Here is a quick lesson in **right triangle trigonometry** for those of you that have forgotten.

Let’s begin by focusing on angle $A$ in the following picture:

![Right Triangle Diagram](image)

Note that the **hypotenuse** is ALWAYS the side opposite the right angle.

The other two sides of the right triangle, called the **legs**, depend on which angle is chosen. In this picture we chose to focus on angle $A$. Therefore the opposite side is $BC$, and the adjacent side is $AC$.

Now you should simply memorize how to compute the six trig functions:

\[
\begin{align*}
\sin A &= \frac{\text{OPP}}{\text{HYP}} \\
\cos A &= \frac{\text{ADJ}}{\text{HYP}} \\
\tan A &= \frac{\text{OPP}}{\text{ADJ}} \\
\csc A &= \frac{\text{HYP}}{\text{OPP}} \\
\sec A &= \frac{\text{HYP}}{\text{ADJ}} \\
\cot A &= \frac{\text{ADJ}}{\text{OPP}}
\end{align*}
\]

Here are a couple of tips to help you remember these:

1. Many students find it helpful to use the word SOHCAHTOA. You can think of the letters here as representing sin, opp, hyp, cos, adj, hyp, tan, opp, adj.

2. The three trig functions on the right are the reciprocals of the three trig functions on the left. In other words, you get them by interchanging the numerator and denominator. It’s pretty easy to remember that the reciprocal of tangent is cotangent. For the other two, just remember that the “s” goes with the “c” and the “c” goes with the “s.” In other words, the reciprocal of sine is cosecant, and the reciprocal of cosine is secant.

To make sure you understand this, compute all six trig functions for each of the angles (except the right angle) in the triangle given in this problem. Please try this yourself before looking at the answers below.
\[ \sin P = \frac{12}{13} \quad \csc P = \frac{13}{12} \quad \sin R = \frac{5}{13} \quad \csc R = \frac{13}{5} \]
\[ \cos P = \frac{5}{13} \quad \sec P = \frac{13}{5} \quad \cos R = \frac{12}{13} \quad \sec R = \frac{13}{12} \]
\[ \tan P = \frac{12}{5} \quad \cot P = \frac{5}{12} \quad \tan R = \frac{5}{12} \quad \cot R = \frac{12}{5} \]

10. Let \( x = \cos \theta \) and \( y = \sin \theta \) for any real value \( \theta \). Then \( x^2 + y^2 = \)

   (A) \(-1\)
   (B) 0
   (C) 1
   (D) It cannot be determined from the information given

* **Solution using a Pythagorean identity:**

\[ x^2 + y^2 = (\cos \theta)^2 + (\sin \theta)^2 = 1 \]

This is choice (C).

**Notes:**
(1) \((\cos \theta)^2\) is usually abbreviated as \(\cos^2 \theta\).
Similarly, \((\sin \theta)^2\) is usually abbreviated as \(\sin^2 \theta\).
In particular, \((\cos \theta)^2 + (\sin \theta)^2\) would be written as \(\cos^2 \theta + \sin^2 \theta\).
(2) One of the most important trigonometric identities is the Pythagorean Identity which says

\[ \cos^2 x + \sin^2 x = 1. \]

11. A line with slope \(\frac{2}{3}\) is translated up 5 units and right 1 unit. What is the slope of the new line?

* Any translation of a line is parallel to the original line and therefore has the same slope. The new line therefore has a slope of \(\frac{2}{3}\).

**Notes:**
(1) If we only moved *some* of the points on the line, then the slope might change. But here we are moving all points on the line simultaneously. Therefore the exact shape and orientation of the line are preserved.
(2) We could also grid in one of the decimals \(0.666\) or \(0.667\).
(3) If the solution is not clear, it is recommended that you draw a picture. Start by drawing a line with slope \(\frac{2}{3}\). One way to do this would be to plot points at \((0,0)\) and \((3,2)\) and then draw a line through these two points.
Now take those same two points and move them up 5 units and right 1 unit to the points (1,5) and (4,7). Draw a line through these two points.

Note that the two lines are parallel.

(4) Recall that the formula for the slope of a line is

\[
\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Let’s verify that the slopes of the two lines mentioned in note (3) are the same.

For the line passing through (0,0) and (3,2), the slope is \(\frac{2-0}{3-0} = \frac{2}{3}\), and for the line passing through (1,5) and (4,7), the slope is \(\frac{7-5}{4-1} = \frac{2}{3}\).

So we see that the two slopes are equal.

(5) **Parallel lines always have the same slope.**

12. In the figure above, adjacent sides meet at right angles and the lengths given are in inches. What is the perimeter of the figure, in inches?

* **Solution by moving the sides of the figure around:** Recall that to compute the perimeter of the figure we need to add up the lengths of all 8 line segments in the figure. We “move” the two smaller vertical segments to the right, and each of the smaller horizontal segments up or down as shown below.
Note that the “bold” length is equal to the “dashed” length. We get a rectangle with length 30 and width 15. Thus, the perimeter is 

\[(2)(30) + (2)(15) = 60 + 30 = 90.\]

**Warning:** Although lengths remain unchanged by moving line segments around, areas will be changed. This method should **not** be used in problems involving areas.

**LEVEL 1: PASSPORT TO ADVANCED MATH**

13. If \(2x^2 - 11 = 5 - 2x^2\), what are all possible values of \(x\) ?

(A) 2 only  
(B) \(-2\) only  
(C) 0 only  
(D) 2 and \(-2\) only

**Solution by plugging in the answer choices:** According to the answer choices we need only check 0, 2, and \(-2\).

\[
x = 0: \quad 2(0)^2 - 11 = 5 - 2(0)^2 \quad -11 = 5 \quad \text{False}
\]

\[
x = 2: \quad 2(2)^2 - 11 = 5 - 2(2)^2 \quad -3 = -3 \quad \text{True}
\]

\[
x = -2: \quad 2(-2)^2 - 11 = 5 - 2(-2)^2 \quad -3 = -3 \quad \text{True}
\]

So the answer is choice (D).

**Notes:**
(1) Since all powers of \(x\) in the given equation are even, 2 and \(-2\) must give the same answer. So we didn’t really need to check \(-2\).

(2) Observe that when performing the computations above, the proper order of operations was followed. Exponentiation was done first, followed by multiplication, and then subtraction was done last.
For example, we have $2(2)^2 - 11 = 2 \cdot 4 - 11 = 8 - 11 = -3$ and $5 - 2(2)^2 = 5 - 2 \cdot 4 = 5 - 8 = -3$.

**Order of Operations:** Here is a quick review of order of operations.

<table>
<thead>
<tr>
<th>P</th>
<th>Parentheses</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Exponentiation</td>
</tr>
<tr>
<td>M</td>
<td>Multiplication</td>
</tr>
<tr>
<td>D</td>
<td>Division</td>
</tr>
<tr>
<td>A</td>
<td>Addition</td>
</tr>
<tr>
<td>S</td>
<td>Subtraction</td>
</tr>
</tbody>
</table>

Note that multiplication and division have the same priority, and addition and subtraction have the same priority.

* **Algebraic solution:** We add $2x^2$ to each side of the given equation to get $4x^2 - 11 = 5$. We then add 11 to get $4x^2 = 5 + 11 = 16$. Dividing each side of this last equation by 4 gives $x^2 = \frac{16}{4} = 4$. We now use the square root property to get $x = \pm 2$. So the answer is choice (D).

**Notes:**
1. The equation $x^2 = 4$ has two solutions: $x = 2$ and $x = -2$. A common mistake is to forget about the negative solution.
2. The square root property says that if $x^2 = c$, then $x = \pm \sqrt{c}$.

This is different from taking the positive square root of a number. For example, $\sqrt{4} = 2$, whereas the equation $x^2 = 4$ has two solutions $x = \pm 2$.

3. Another way to solve the equation $x^2 = 4$ is to subtract 4 from each side of the equation, and then factor the difference of two squares as follows:

$$x^2 - 4 = 0$$
$$x^2 - 4 = 0$$
$$(x - 2)(x + 2) = 0$$

We now set each factor equal to 0 to get $x - 2 = 0$ or $x + 2 = 0$.

So $x = 2$ or $x = -2$.

**14.** A function $g(x)$ is defined as $g(x) = -5x^2$. What is $g(-2)$?

(A) $-100$
(B) $-20$
(C) $20$
(D) $50$
* \( g(-2) = -5(-2)^2 = -5(4) = -20 \), choice (B).

**Notes:**
1. The variable \( x \) is a placeholder. We evaluate the function \( g \) at a specific value by substituting that value in for \( x \). In this question we replaced \( x \) by \(-2\).

2. The exponentiation was done first, followed by the multiplication. See the end of the solution to problem 13 for more information on order of operations.

3. To square a number means to multiply it by itself. So \((-2)^2 = (-2)(-2) = 4\).

4. We can do the whole computation in our calculator (if a calculator is allowed for the problem) in one step. Simply type \(-5(-2)^2\) ENTER. The output will be \(-20\).

Make sure to use the minus sign and not the subtraction symbol. Otherwise the calculator will give an error.

15. A system of three equations in two unknowns and their graphs in the \( xy \)-plane are shown above. How many solutions does the system have?

   (A) None  
   (B) Two  
   (C) Four  
   (D) Six

* **Solution by looking at the graph:** There is no point that is common to all three graphs. So the system has no solutions, choice (A).
Notes: (1) A solution to the system of equations is a point that satisfies all three equations simultaneously. Graphically this means that the point is on all three graphs. Although there are several points that are common to two of the graphs, there are none that are common to all three.

(2) The graph of the equation \(x^2 - y^2 = 9\) is the hyperbola in the figure above with vertices \((-3,0)\) and \((3,0)\).

(3) The graph of the equation \(\frac{x^2}{16} + \frac{y^2}{4} = 1\) is the ellipse in the figure above with vertices \((-4,0)\), \((4,0)\), \((0,2)\), and \((0,-2)\).

(4) The graph of the equation \(x + 2y = 4\) is the line in the figure above with intercepts \((4,0)\) and \((0,2)\).

\((4,0)\) is the \textit{x-intercept} of the line, and \((0,2)\) is the \textit{y-intercept} of the line.

(5) Consider the following system of equations:

\[
\begin{align*}
\frac{x^2}{16} + \frac{y^2}{4} &= 1 \\
x + 2y &= 4
\end{align*}
\]

This system has the two solutions \((0,2)\) and \((4,0)\). These are the two points common to the graphs of these two equations (the ellipse and the line), also known as points of intersection of the two graphs.

(6) Consider the following system of equations:

\[
\begin{align*}
x^2 - y^2 &= 9 \\
x + 2y &= 4
\end{align*}
\]

This system also has two solutions. These are the two points common to the hyperbola and the line. Finding these two solutions requires solving the system algebraically, which we won’t do here. One of these solutions can be seen on the graph. It looks to be approximately \((3.1,0.5)\). The second solution does not appear on the portion of the graph that is displayed. If we continued to graph the line and hyperbola to the left we would see them intersect one more time.

(6) Consider the following system of equations:

\[
\begin{align*}
x^2 - y^2 &= 9 \\
\frac{x^2}{16} + \frac{y^2}{4} &= 1
\end{align*}
\]
This system has four solutions. These are the four points common to the hyperbola and the ellipse. Finding these four solutions requires solving the system algebraically, which we won’t do here. These solutions can be seen clearly on the graph.

**Algebraic solution:** Observe from the graph that the points (0,2) and (4,0) are intersection points of the line and the ellipse. In other words they are solutions to the following system:

\[
\begin{align*}
x^2 + \frac{y^2}{4} &= 1 \\
x + 2y &= 4
\end{align*}
\]

We can verify this by substituting each point into each equation.

(0,2):

\[
\begin{align*}
x^2 + \frac{y^2}{4} &= 1 \\x &= 0 \Rightarrow 0 + 2(2) = 4 \Rightarrow 4 = 4
\end{align*}
\]

(4,0):

\[
\begin{align*}
x^2 + \frac{y^2}{4} &= 1 \\x &= 4 \Rightarrow 4 + 2(0) = 4 \Rightarrow 4 = 4
\end{align*}
\]

When we plug each of these points into the equation for the hyperbola however, we get the following:

(0,2): 

\[
x^2 - y^2 = 9 \Rightarrow 0^2 - 2^2 = 9 \Rightarrow -4 = 9
\]

(4,0): 

\[
x^2 - y^2 = 9 \Rightarrow 4^2 - 0^2 = 9 \Rightarrow 16 = 9
\]

Since we wound up with false equations, neither of these points are on the hyperbola.

It follows that the system of equations has no solutions, choice (A).

**Notes:** (1) Although I do not recommend this for this problem, we can solve the following system formally using the substitution method.

\[
\begin{align*}
x^2 + \frac{y^2}{4} &= 1 \\
x + 2y &= 4
\end{align*}
\]

Let’s begin by solving the second equation for \(x\) by subtracting 2\(y\) from each side of the equation to get \(x = 4 - 2y\).

We now replace \(x\) by \(4 - 2y\) in the first equation and solve for \(y\).

\[
\begin{align*}
\frac{x^2}{16} + \frac{y^2}{4} &= 1 \\
\frac{(4-2y)^2}{16} + \frac{y^2}{4} &= 1
\end{align*}
\]
We multiply each side of this last equation by 16 to get

\[(4 - 2y)^2 + 4y^2 = 16\]

Now \((4 - 2y)^2 = (4 - 2y)(4 - 2y) = 16 - 8y - 8y + 4y^2\). So we have

\[16 - 8y - 8y + 4y^2 + 4y^2 = 16\]

We cancel the 16 from each side and combine like terms on the left to get

\[-16y + 8y^2 = 0\]

We factor \(-8y\) and note that \(-\frac{16y}{-8y} = 2\) and \(\frac{8y^2}{-8y} = -y\) to get

\[-8y(2 - y) = 0\]

We now set each factor equal to zero.

\[-8y = 0 \quad \text{or} \quad 2 - y = 0\]

So we get the two solutions \(y = 0\) and \(y = 2\).

We can now substitute these \(y\)-values into either equation. Let’s use the equation of the line since it’s simpler:

\[y = 0: x + 2y = 4 \iff x + 2(0) = 4 \iff x = 4\]
\[y = 2: x + 2y = 4 \iff x + 2(2) = 4 \iff x + 4 = 4 \iff x = 0\]

So we see that the two points of intersection of the ellipse and the line are \((4,0)\) and \((0,2)\).

(2) If we wanted to find the intersection points of the line and the hyperbola we would solve the following system as we did in note (1):

\[x^2 - y^2 = 9\]
\[x + 2y = 4\]

In this case however the algebra will be much messier and the solutions do not “look very nice.”

It will never be necessary to do such messy algebra on the SAT, so we leave this an optional exercise for the interested reader.

Similarly for the intersection points of the hyperbola and the ellipse we would solve the following system:

\[x^2 - y^2 = 9\]
\[\frac{x^2}{16} + \frac{y^2}{4} = 1\]
Again, the algebra here is messy, and we leave this as an optional exercise.

16. Which of the following graphs could not be the graph of a function?

* Only choice (D) fails the **vertical line test**. In other words, we can draw a vertical line that hits the graph more than once:

So the answer is choice (D).

\[
\begin{align*}
f(x) &= 5x + 3 \\
g(x) &= x^2 - 5x + 2
\end{align*}
\]

17. The functions \(f\) and \(g\) are defined above. What is the value of \(f(10) - g(5)\)?

* We have

\[
\begin{align*}
f(10) &= 5(10) + 3 = 50 + 3 = 53 \\
g(5) &= 5^2 - 5(5) + 2 = 25 - 25 + 2 = 2
\end{align*}
\]

Therefore \(f(10) - g(5) = 53 - 2 = 51\).
18. The table above gives some values of the functions $p$, $q$, and $r$. At which value of $x$ does $q(x) = p(x) + r(x)$?

**Solution by guessing:** The answer is an integer between 1 and 5 inclusive (these are the $x$-values given). So let’s start with $x = 3$ as our first guess. From the table $p(3) = -4$, $q(3) = -7$, and $r(3) = 3$. Therefore $p(3) + r(3) = -4 + 3 = -1$. This is not equal to $q(3) = -7$ so that 3 is not the answer.

Let’s try $x = 4$ next. From the table $p(4) = -5$, $q(4) = -7$, and $r(4) = -2$. So $p(4) + r(4) = -5 + (-2) = -7 = q(4)$.

Therefore the answer is 4.

*Quick solution:* We can just glance at the rows quickly and observe that in the row corresponding to $x = 4$, we have $-5 + (-2) = -7$. Thus, the answer is 4.
Questions 19 - 21 refer to the following information.

Ten 25 year old men were asked how many hours per week they exercise and their resting heart rate was taken in beats per minute (BPM). The results are shown as points in the scatterplot below, and the line of best fit is drawn.

19. How many of the men have a resting heart rate that differs by more than 5 BPM from the resting heart rate predicted by the line of best fit?

   (A) None  
   (B) Two  
   (C) Three  
   (D) Four  

* The points that are more than 5 BPM away from the line of best fit occur at 1, 4, and 8 hours. So there are Three of them, choice (C).

Notes: (1) One of the two men that exercise 1 hour per week has a resting heart rate of approximately 68 BPM. The line of best fit predicts approximately 77 BPM. So this difference is $77 - 68 = 9$ BPM.

Similarly, at 4 we have a difference of approximately $75 - 67 = 8$ BPM, and at 8 we have a difference of approximately $60 - 54 = 6$ BPM.
(2) At 5, the point below the curve corresponds to a heart rate that differs from that predicted by the line of best fit by approximately $64 - 59 = 5$ BPM. Since this is not more than 5, we do not include this point in the count.

20. Based on the line of best fit, what is the predicted resting heart rate for someone that exercises three and a half hours per week?

(A) 66 BPM  
(B) 68 BPM  
(C) 70 BPM  
(D) 72 BPM

* The point $(3.5, 68)$ seems to be on the line of best fit. So the answer is 68 BPM, choice (B).

21. Which of the following is the best interpretation of the slope of the line of best fit in the context of this problem?

(A) The predicted number of hours that a person must exercise to maintain a resting heart rate of 50 BPM.
(B) The predicted resting heart rate of a person that does not exercise.
(C) The predicted decrease in resting heart rate, in BPM, for each one hour increase in weekly exercise.
(D) The predicted increase in the number of hours of exercise needed to increase the resting heart rate by one BPM.

* The slope of the line is the \( \frac{\text{change in predicted heart rate}}{\text{change in hours of exercise}} \). If we make the denominator a 1 hour increase, then the fraction is the change in predicted heart rate per 1 hour increase. Since the line is moving downward from left to right, we can replace “change” in the numerator by “decrease.” So the answer is choice (C).

*Note: Recall that the slope of a line is

\[
\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in vertical distance}}{\text{change in horizontal distance}}
\]

In this problem the change in vertical distance is the change in resting heart rate, in BPM, and the change in horizontal distance is the change in hours of exercise per week.
22. The mean annual salary of an NBA player, \( S \), can be estimated using the equation \( S = 161,400(1.169)^t \), where \( S \) is measured in thousands of dollars, and \( t \) represents the number of years since 1980 for \( 0 \leq t \leq 20 \). Which of the following statements is the best interpretation of 161,400 in the context of this problem?

(A) The estimated mean annual salary, in dollars, of an NBA player in 1980.
(B) The estimated mean annual salary, in dollars, of an NBA player in 2000.
(C) The estimated yearly increase in the mean annual salary of an NBA player.
(D) The estimated yearly decrease in the mean annual salary of an NBA player.

* When \( t = 0 \), we have 
\[
S = 161,400(1.169)^0 = 161,400(1) = 161,400.
\]

Since \( t = 0 \) corresponds to the year 1980, it follows that 161,400 is the estimated mean annual salary, in dollars, of an NBA player in 1980. This is choice (A).

Notes: (1) The year 2000 corresponds with \( t = 20 \). So the estimated mean annual salary, in dollars, of an NBA player in 2000 would be 
\[
S = 161,400(1.169)^{20}.
\]
This is a number much larger than 161,400 (it is approximately 3,666,011).

(2) The function given in this problem is an exponential function. In general, exponential functions have the form \( y = ab^t \). Note that \( t = 0 \) corresponds to \( y = a \). In other words, the initial amount is always \( a \).

In this problem \( t = 0 \) corresponds to the year 1980, and so the 161,400 gives the mean annual salary in 1980.

Unlike a linear function, an exponential function does not have a constant slope. So in this problem the yearly increase or decrease in mean annual salary cannot be described by a single number.

(3) Let’s compare this to the analogous linear function. Suppose for a moment that the equation given instead was 
\[
S = 1.169t + 161,400
\]
In this case, the number 161,400 would still describe the estimated mean annual salary, in dollars, of an NBA player in 1980.

The number 1.169 would describe the estimated yearly increase in the mean annual salary of an NBA player.

23. A biologist was interested in the number of times a field cricket chirps each minute on a sunny day. He randomly selected 100 field crickets from a garden, and found that the mean number of chirps per minute was 112, and the margin of error for this estimate was 6 chirps. The biologist would like to repeat the procedure and attempt to reduce the margin of error. Which of the following samples would most likely result in a smaller margin of error for the estimated mean number of times a field cricket chirps each minute on a sunny day?

(A) 50 randomly selected crickets from the same garden.
(B) 50 randomly selected field crickets from the same garden.
(C) 200 randomly selected crickets from the same garden.
(D) 200 randomly selected field crickets from the same garden.

* Increasing the sample size while keeping the population the same will most likely decrease the margin of error. So the answer is choice (D).

Notes: (1) Decreasing the sample size will increase the margin of error. This allows us to eliminate choices (A) and (B).

(2) The original sample consisted of only field crickets. If we were to allow the second sample to include all crickets, then we have changed the population. We cannot predict what impact this would have on the mean and margin of error. This allows us to eliminate choice (C).

Technical note: In reality there is a correlation between the frequency of cricket chirps and temperature. You can estimate the current temperature, in degrees Fahrenheit, by counting the number of times a cricket chirps in 15 seconds and adding 37 to the result.
24. A survey was conducted among a randomly chosen sample of 250 single men and 250 single women about whether they owned any dogs or cats. The table below displays a summary of the survey results.

<table>
<thead>
<tr>
<th></th>
<th>Dogs Only</th>
<th>Cats Only</th>
<th>Both</th>
<th>Neither</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>92</td>
<td>14</td>
<td>18</td>
<td>126</td>
<td>250</td>
</tr>
<tr>
<td>Women</td>
<td>75</td>
<td>42</td>
<td>35</td>
<td>98</td>
<td>250</td>
</tr>
<tr>
<td>Total</td>
<td>167</td>
<td>56</td>
<td>53</td>
<td>224</td>
<td>500</td>
</tr>
</tbody>
</table>

What fraction of the people surveyed who said they own dogs are women?

* There are 75 + 35 = 110 women who said they own dogs, and there are a total of 167 + 53 = 220 people who said they own dogs. Therefore, the fraction of reported dog owners that are women is \( \frac{110}{220} = \frac{1}{2} \) or .5.

**Notes:** (1) There are two columns that represent people who said they own dogs: the column labeled “Dogs Only,” and the column labeled “Both.”